

e-voting

Seminar Advanced Topics in Cryptography

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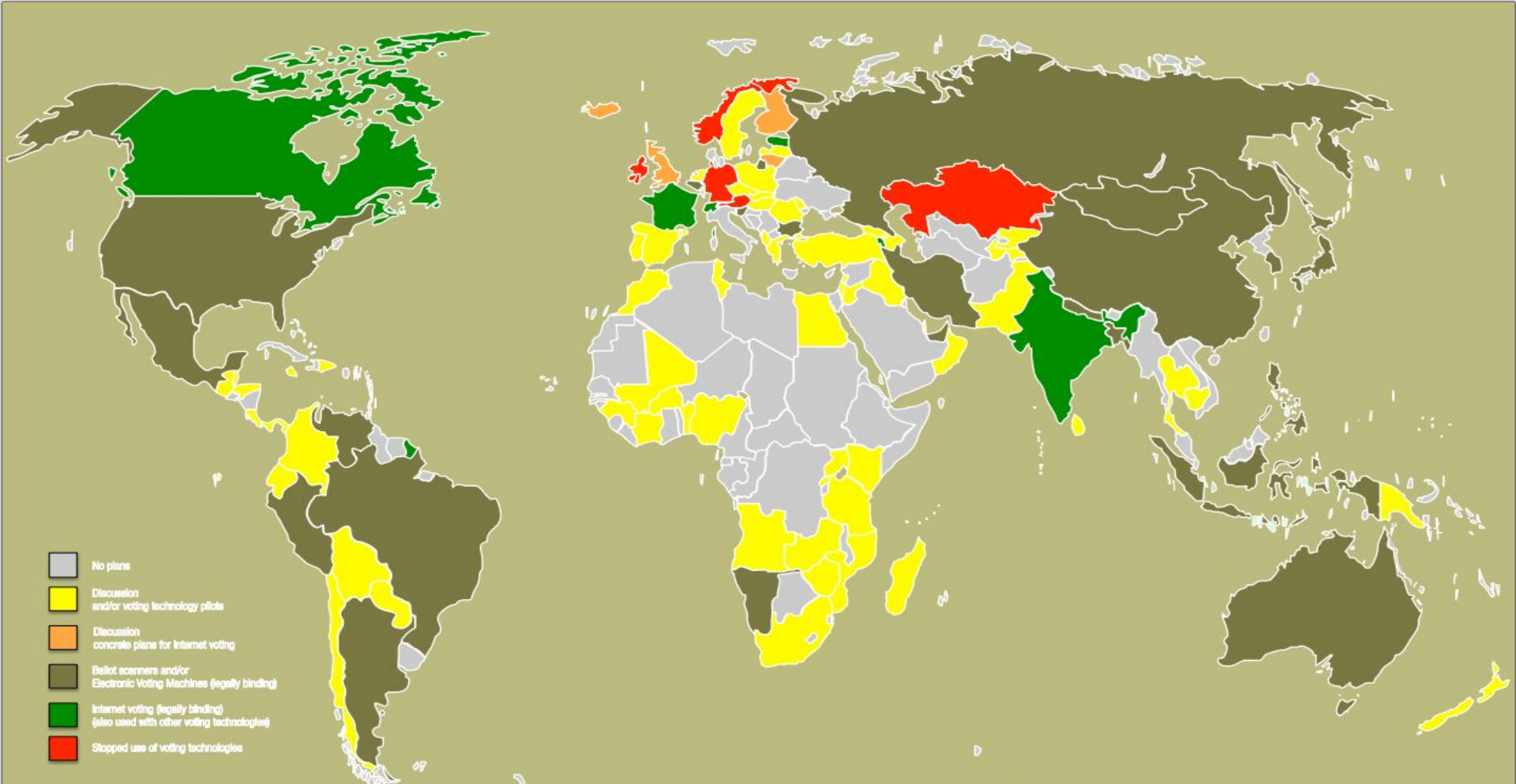
e-voting

- Introduction
- Authentication in Electronic Elections
- Security Aspects
- Bullet Points From Paper
 - Authentication with Weaker Trust Assumptions for Voting Systems(Quaglia & Smyth)

Introduction

e-voting

- Decision making process
- Hard, conflicting security requirements for remote voting:
 - Integrity
 - Confidentiality



- No plans
- Discussion and/or voting technology pilots
- Discussion concrete plans for Internet voting
- Ballot scanners and/or Electronic Voting Machines (legally binding)
- Internet voting (legally binding) (also used with other voting technologies)
- Stopped use of voting technologies

World Map of Electronic Voting 5

Authentication in Electronic Elections

External Authentication*

*Helios(via Facebook, Google), Yahoo(via OAuth)

$$\Gamma_{\text{Ext}} = (\text{Setup}, \text{Vote}, \text{Tally}, \text{Verify})$$

Identities: Tallier (T), Voter (V)

$$\underline{T}: (pk, sk, mb, mc) \leftarrow \text{Setup}(\kappa)$$

$$\underline{V}: b \text{ or } \perp \leftarrow \text{Vote}(pk, nc, v, \kappa)$$

$$\underline{T}: (V, pf) \leftarrow \text{Tally}(sk, nc, bb, \kappa)$$

$$\underline{V}: s \leftarrow \text{Verify}(pk, nc, bb, V, pf, \kappa)$$

*K: security parameter,
pk: public key of tallier,
sk: secret key of tallier*

*mb: max. #ballots,
mc: max. #candidates,
pd: public credential,*

*d: private credential,
nc: some #candidates,
v: voter's vote,*

Internal Authentication**

** Voting system by Juels, Catalano & Jakobsson via cryptographic primitives

$$\Gamma_{\text{Int}} = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$$

Identities: Tallier (T), **Registrar (R)**, Voter (V)

$$\underline{T}: (pk, sk, mb, mc) \leftarrow \text{Setup}(\kappa)$$

$$\underline{R}: (pd, d) \leftarrow \text{Register}(pk, \kappa)$$

$$\underline{V}: b \leftarrow \text{Vote}(d, pk, nc, v, \kappa)$$

$$\underline{T}: (V, pf) \leftarrow \text{Tally}(sk, nc, bb, L, \kappa)$$

$$\underline{V}: s \leftarrow \text{Verify}(pk, nc, bb, L, V, pf, \kappa)$$

*b: ballot,
bb: bulletin board,
L: electoral roll,*

*pf: non-interactive proof,
V: election outcome vector, 7
s: election successful bit $\in \{0, 1\}$*

Correctness

$(pk, sk, mb, mc) \leftarrow \text{Setup}(\kappa)$

for $1 \leq i \leq nb$ do

$(pd_i, d_i) \leftarrow \text{Register}(pk, \kappa)$

$b_i \leftarrow \text{Vote}(\langle d_i \rangle, pk, nc, v_i, \kappa)$

$V[v_i] \leftarrow V[v_i] + 1$

$(V', pf) \leftarrow \text{Tally}(sk, nc, \{b_1, \dots, b_{nb}\}, \langle \{pd_1, \dots, pd_{nb}\} \rangle, \kappa)$

$$\text{prob}(V = V' \mid nb \leq mb \wedge nc \leq mc) > 1 - \text{negl}(\kappa)$$

Security Aspects

A white thought bubble with a black outline, containing the text 'Smyth, Frink & Clarkson'. The bubble has a small tail pointing downwards and to the left.

Smyth, Frink
& Clarkson

Security Aspects

Security Aspects

- Ballot secrecy
- Election verifiability
 - Individual verifiability
 - Universal verifiability
- Eligibility verifiability

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- Ballot secrecy
- Election verifiability
 - Individual verifiability
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External Ballot Secrecy Game $G^{\text{Bal-Sec-Ext}}$

- Run election setup $(pk, sk, mb, mc) \leftarrow \text{Setup}(\kappa)$.
- Call the attacker A with input 1^κ and pk . Await a number nc .
- Set $B \leftarrow \emptyset$.
- Choose a hidden bit $h \leftarrow \{0,1\}$ randomly.
- Prepare a secrecy oracle O^{Sec} . When called with $v_0, v_1 \in \{1, \dots, nc\}$, the oracle creates ballot $b \leftarrow \text{Vote}(pk, nc, v_h, \kappa)$, adds it to $B \leftarrow B \cup \{(b, v_0, v_1)\}$ and returns b .
- Call the attacker A with O^{Sec} . Await a bb .
- Run tally $(V, pf) \leftarrow \text{Tally}(sk, nc, bb, \kappa)$.
- Call the attacker A with input V and pf . Await a guess $h' \in \{0,1\}$.
- If $h = h' \wedge \text{balanced}(bb, nc, B) \wedge 1 \leq nc \leq mc \wedge \|bb\| \leq mb$ then **ACCEPT**
else **REJECT**.

*balanced(bb, nc, B):

$\forall v \in \{1, \dots, nc\}$ we have

$$|\{b \mid b \in bb \wedge \exists v_1. (b, v, v_1) \in B\}| = |\{b \mid b \in bb \wedge \exists v_0. (b, v_0, v) \in B\}|$$

Definition

An electronic election scheme with external auth.

$\Gamma_{\text{Ext}} = (\text{Setup}, \text{Vote}, \text{Tally}, \text{Verify})$

satisfies Ballot-Secrecy-Ext

iff

for each ppt attacker A the advantage

$$\text{adv}^{\text{Bal-Sec-Ext}}(A) = |\text{prob}(G^{\text{Bal-Sec-Ext}}(A) = \text{ACCEPT}) - 1/2|$$

is at most $\text{negl}(\kappa)$.

Internal Ballot Secrecy Game $G^{\text{Bal-Sec-Int}}$

- Run election setup $(pk, sk, mb, mc) \leftarrow \text{Setup}(\kappa)$.
- **Call the attacker A with input 1^κ and pk . Await a number nv .**
- **for $1 \leq i \leq nv$ do**
 $(pd_i, d_i) \leftarrow \text{Register}(pk, \kappa)$.
- Call the attacker A with input $\{pd_1, \dots, pd_{nv}\}$. Await a number nc .
- Set $B \leftarrow \emptyset$, $R \leftarrow \emptyset$.
- Choose a hidden bit $h \leftarrow \{0,1\}$ randomly.
- **Prepare a secrecy oracle O^{Sec} . When called with i , adds i to R and returns d_i if $i \notin R$. When called with $i \notin R$ and $v_{op} v_1 \in \{1, \dots, nc\}$, the oracle creates ballot $b \leftarrow \text{Vote}(d_i, pk, nc, v_{op}, \kappa)$ and adds it to $B \leftarrow B \cup \{(b, v_{op} v_1)\}$, adds i to R and returns b .**
- Call the attacker A with O^{Sec} . Await a bb .
- Run tally $(V, pf) \leftarrow \text{Tally}(sk, nc, bb, \{pd_1, \dots, pd_{nv}\}, \kappa)$.
- Call the attacker A with input V and pf . Await a guess $h' \in \{0,1\}$.
- If $h = h' \wedge \text{balanced}(bb, nc, B) \wedge 1 \leq nc \leq mc \wedge \|bb\| \leq mb$ then
 ACCEPT
 else **REJECT**.

Definition

An electronic election scheme with internal auth.

$\Gamma_{\text{Int}} = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$

satisfies Ballot-Secrecy-Int

iff

for each ppt attacker A the advantage

$$\text{adv}^{\text{Bal-Sec-Int}}(A) = |\text{prob}(G^{\text{Bal-Sec-Int}}(A) = \text{ACCEPT}) - 1/2|$$

is at most $\text{negl}(\kappa)$.

Security Aspects

- Ballot secrecy
- Election verifiability
 - Individual verifiability
 - Universal verifiability
- Eligibility verifiability

External Individual Verifiability Game

$G^{\text{IV-Ext}}$

- Call the attacker A with input 1^κ . Await pk, nc, v, v' .
- Run vote algorithm for v and v' :
 - $b \leftarrow \text{Vote}(pk, nc, v, \kappa)$
 - $b' \leftarrow \text{Vote}(pk, nc, v', \kappa)$
- If $b = b' \wedge b \neq \perp \wedge b' \neq \perp$ then **ACCEPT**
else **REJECT**.

Definition

An electronic election scheme with external auth.

$\Gamma_{\text{Ext}} = (\text{Setup}, \text{Vote}, \text{Tally}, \text{Verify})$

satisfies IV-Ext

iff

for each ppt attacker A the advantage

$$\text{adv}^{\text{IV-Ext}}(A) = |\text{prob}(G^{\text{IV-Ext}}(A) = \text{ACCEPT})|$$

is at most $\text{negl}(\kappa)$.

Internal Individual Verifiability Game

$G^{\text{IV-Int}}$

- Call the attacker A with input 1^κ . Await **pk and nv** .
- **for $1 \leq i \leq nv$ do**
 $(pd_i, d_i) \leftarrow \text{Register}(pk, \kappa)$.
- **Let $L \leftarrow \{pd_1, \dots, pd_{nv}\}$ and $\text{Crypt} \leftarrow \emptyset$.**
- **Prepare oracle O^{IV} . When called with $i \in \{1, \dots, nv\}$, adds d_i to Crypt and returns d_i .**
- **Call the attacker A with L and O^{IV} . Await nc, v, v', i, j .**
- Run vote algorithm for v and v' :
 $b \leftarrow \text{Vote}(d_i, pk, nc, v, \kappa)$
 $b' \leftarrow \text{Vote}(d_j, pk, nc, v', \kappa)$
- If $b = b' \wedge b \neq \perp \wedge b' \neq \perp \wedge i \neq j \wedge d_i \notin \text{Crypt} \wedge d_j \notin \text{Crypt}$
 then **ACCEPT** else **REJECT**.

Definition

An electronic election scheme with internal auth.

$\Gamma_{\text{Int}} = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$

satisfies IV-Int

iff

for each ppt attacker A the advantage

$$\text{adv}^{\text{IV-Int}}(A) = |\text{prob}(G^{\text{IV-Int}}(A) = \text{ACCEPT})|$$

is at most $\text{negl}(\kappa)$.

Security Aspects

- Ballot secrecy
- Election verifiability
 - Individual verifiability
 - Universal verifiability
- Eligibility verifiability

Algorithm Verify is required to accept iff *the election outcome is correct*.

- The outcome vector length must be nc .
- Component β of Tally outcome vector equals ℓ iff there exist ℓ unique ballots on the bulletin board that are votes for candidate β .
- The output represents the choices used to construct the recorded ballots.

Algorithm Verify is required to accept iff *the election outcome is correct*.

Injectivity

Ballots interpreted
only for one
candidate.

$(v \neq v' \Rightarrow b \neq b')$

Algorithm **Verify** is required to accept iff *the election outcome is correct*.

Injectivity

Ballots interpreted only for one candidate.

Completeness

Tally produces election outcomes that will be accepted by **Verify**.

$(\text{pr}[|bb| \leq mb \wedge nc \leq mc \Rightarrow \text{Verify}()=1] > 1-\text{negl}())$

Algorithm **Verify** is required to accept iff *the election outcome is correct*.

Injectivity

Ballots interpreted only for one candidate.

Completeness

Tally produces election outcomes that will be accepted by **Verify**.

Soundness

The probability to conduct a scenario where **Verify** accepts although the election outcome is not correct is negligible.

$$\Pr[V^* \neq V \Rightarrow \text{Verify}(V^*) = 1] \leq \text{negl}()$$

$\Gamma_{\text{Ext/Int}} = (\text{Setup}, \langle \text{Register} \rangle, \text{Vote}, \text{Tally}, \text{Verify})$ satisfies
Universal Verifiability (UV-Ext/Int)

if

Injectivity, Completeness and Soundness are satisfied.

Security Aspects

- Ballot secrecy
- Election verifiability
 - Individual verifiability
 - Universal verifiability
- Eligibility verifiability

Eligibility Verifiability Game $G^{\text{EV-Int}}$

- Call the attacker A with input 1^κ . Await pk and nv .
- for $1 \leq i \leq nv$ do
 $(pd_i, d_i) \leftarrow \text{Register}(pk, \kappa)$.
- Let $L \leftarrow \{pd_1, \dots, pd_{nv}\}$, $\text{Crpt} \leftarrow \emptyset$, and $\text{Rvld} \leftarrow \emptyset$.
- Prepare oracle O^{EV} . When called with i, v, nc ;
 computes $b \leftarrow \text{Vote}(d_i, pk, nc, v, \kappa)$, adds b to Rvld and outputs b .
- Prepare oracle O^{IV} . When called with $i \in \{1, \dots, nv\}$, adds d_i to Crpt
 and returns d_i .
- Call the attacker A with L, O^{EV} and O^{IV} . Await nc, v, i, b .
- If $b \neq \perp \wedge b \in \text{Rvld} \wedge d_i \in \text{Crpt} \wedge \exists r: b = \text{Vote}(d_i, pk, nc, v, \kappa; r)$
 then **ACCEPT** else **REJECT**.

Definition

An electronic election scheme with internal auth.

$\Gamma_{\text{Int}} = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$

satisfies EV-Int

iff

for each ppt attacker A the advantage

$$\text{adv}^{\text{EV-Int}}(A) = |\text{prob}(G^{\text{EV-Int}}(A) = \text{ACCEPT})|$$

is at most $\text{negl}(\kappa)$.

Authentication with Weaker Trust Assumptions for Voting Systems*

(*)

Elizabeth A. Quaglia and Ben Smyth (2018)
<https://eprint.iacr.org/2018/222.pdf>

Ext2Int

- $\Gamma_{\text{Ext}} \xrightarrow{\substack{\text{+digital signature} \\ \text{+ NIPS}}} \Gamma_{\text{Int}}$
- Relation $R(\Gamma, \Omega)$ such that
 $((pk, b, \sigma, nc, \kappa), (v, r, d, r')) \in R(\Gamma, \Omega)$
 \Leftrightarrow
 $b = \text{Vote}(pk, nc, v, \kappa; r) \wedge \sigma = \text{Sign}_{\Omega}(d, b; r')$
- $\text{FS}(\Sigma, H) = (\text{Prove}_{\Sigma}, \text{Verify}_{\Sigma})$
- $\Omega = (\text{Gen}_{\Omega}, \text{Sign}_{\Omega}, \text{Verify}_{\Omega})$
- $\text{Ext2Int}(\Gamma, \Omega, \Sigma, H)$ where
 Γ : Underlying election scheme
 Ω : Signature Scheme
 Σ : Sigma Protocol for R
 H : Hash Function

Construction

$\text{Ext2Int}(\Gamma, \Omega, \Sigma, H) = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$
such that:

$\text{Setup}(\kappa): (pk, sk, mb, mc) \leftarrow \text{Setup}_{\Gamma}(\kappa)$

$\text{Register}(pk, \kappa): (pd, (pd, d)) \leftarrow \text{Gen}_{\Omega}(pk)$

$\text{Vote}(d', pk, nc, v, \kappa)$: if $\text{parse}(d') = (pd, d)$ fails then \perp else
pick r, r' at random and compute:
 $b \leftarrow \text{Vote}_{\Gamma}(pk, nc, v, \kappa; r)$
 $\sigma \leftarrow \text{Sign}_{\Omega}(d, b; r')$
 $\tau \leftarrow \text{Prove}_{\Sigma}((pk, b, \sigma, nc, \kappa), (v, r, d, r'), \kappa)$
and outputs (pd, b, σ, τ) .

$\text{Tally}(sk, nc, bb, L, \kappa): (V, pf) \leftarrow \text{Tally}_{\Gamma}(sk, \text{auth}(bb, L), nc, \kappa)$

$\text{Verify}(pk, nc, bb, L, V, pf, \kappa): s \leftarrow \text{Verify}_{\Gamma}(pk, \text{auth}(bb, L), nc, V, pf, \kappa)$

Ext2Int

$\text{auth}(bb, L) =$

$\{b \mid (pd, b, \sigma, \tau) \in bb \wedge$

$\text{Verify}_\Omega(pd, b, \sigma) = 1 \wedge$

$\text{Verify}_\Sigma((pk, b, nc, \kappa), \tau, \kappa) = 1 \wedge$

$pd \in L \wedge$

$(pd, b', \sigma', \tau') \notin bb \setminus \{(pd, b, \sigma, \tau)\} \wedge$

$\text{Verify}_\Omega(pd, b', \sigma') = 1\}$.

OR

One cannot vote more than once. (Vote once or never)

Σ : Signature Scheme

Σ : Sigma Protocol for R

H: Hash Function

Construction

$\text{Ext2Int}(\Gamma, \Omega, \Sigma, H) = (\text{Setup}, \text{Register}, \text{Vote}, \text{Tally}, \text{Verify})$
such that:

$\text{Setup}(\kappa): (pk, sk, mb, mc) \leftarrow \text{Setup}_\Gamma(\kappa)$

$\text{Register}(pk, \kappa): (pd, (pd, d)) \leftarrow \text{Gen}_\Omega(pk)$

$\text{Vote}(d', pk, nc, v, \kappa):$ if $\text{parse}(d') = (pd, d)$ fails then \perp else

pick r, r' at random computes:

$b \leftarrow \text{Vote}_\Gamma(pk, nc, v, \kappa; r)$

$\sigma \leftarrow \text{Sign}_\Omega(d, b; r')$

$\text{Prove}_\Sigma((pk, b, \sigma, nc, \kappa), (v, r, d, r'), \kappa)$

(pd, b, σ, τ) .

$\text{Tally}(sk, nc, bb, L, v, pf, \kappa): (v, pf) \leftarrow \text{Tally}_\Gamma(sk, \text{auth}(bb, L), nc, \kappa)$

$\text{Verify}(pk, nc, bb, L, v, pf, \kappa): s \leftarrow \text{Verify}_\Gamma(pk, \text{auth}(bb, L), nc, v, pf, \kappa)$

Lemma:

Let Γ be an election scheme with external authentication, Ω be a digital signature scheme, Σ be a sigma protocol for relation $R(\Gamma, \Omega)$, and H be a random oracle.

If

Ω satisfies strong unforgeability,

then

$\text{Ext2Int}(\Gamma, \Omega, \Sigma, H)$ is an election scheme with internal authentication.

Security of Ext2Int

- Let Γ be an election scheme with external authentication, Ω be a digital signature scheme, Σ be a sigma protocol for relation $R(\Gamma, \Omega)$, and H be a random oracle.

If

Γ satisfies *Ballot-Secrecy-Ext*, Σ satisfies *special soundness* and *special honest verifier zero-knowledge*, and Ω satisfies *strong unforgeability*

Then

Election scheme with internal authentication $\text{Ext2Int}(\Gamma, \Omega, \Sigma, H)$ satisfies *Ballot-Secrecy-Int*.

Pf. Sketch: ...

Security of Ext2Int

- Let Γ be an election scheme with external authentication, Ω be a digital signature scheme, Σ be a sigma protocol for relation $R(\Gamma, \Omega)$, and H be a random oracle.

If

Ω satisfies *strong unforgeability*, Σ satisfies *special soundness* and *special honest verifier zero-knowledge*, and Γ satisfies *UV-Ext*

Then

Election scheme with internal authentication $\text{Ext2Int}(\Gamma; \Omega; \Sigma; H)$ satisfies *IV-Int*, *EV-Int*, and *UV-Int*.

Pf. Sketch: ...

Q&A

