The Journey towards a Reference Implementation of IPSec

Automatic Security Analysis with Tamarin-Prover

Eike Stadtländer

July 12, 2018

Outline

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The security proofs are not always trustworthy (Halevi 2005; Bellare and Rogaway 2004). Automatic security analysis aims to improve trustworthiness of security proofs.

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    - messages are not bitstrings but terms
    - relations between terms are given by equational theories

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- Security protocol verification tool
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- Symbolic model
  - messages are not bitstrings but terms
  - relations between terms are given by equational theories
- Dolev-Yao attacker
  - cryptographic primitives are handled as black-boxes
  - active attacker has complete control over the network
  - access to a corrupt oracle

Term Algebras and Cryptographic Messages I

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Example (Cryptographic messages)

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2. For every k-ary function symbol $f : s_1 \times \cdots \times s_k \to s \in \Sigma$, we also have $f : \mathrm{top}(s_1) \times \cdots \times \mathrm{top}(s_k) \to \mathrm{top}(s) \in \Sigma$.

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Then given $A \subseteq \bigcup_{s \in S} \mathcal{C}_s \cup \bigcup_{s \in S} \mathcal{V}_s$, $\mathcal{T}_\Sigma(A)$ denotes the set of all well-sorted terms constructed over $\Sigma \cup A$.

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$$m \in \mathcal{V}_{\mathsf{msg}}, \quad \mathsf{fst}(\langle m, n \rangle), \quad \mathsf{senc}(m, k), \quad \mathsf{sdec}(k_2, \mathsf{senc}(k_1, m))$$

Equational Theories and Cryptographic Primitives

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Example (Cryptographic primitives)

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Given $\mathbf{\Sigma}_{\mathsf{PHS}}$ as before. We define

$$E_{\mathsf{PHS}} = \{\mathsf{fst}(\langle x, y\rangle) = x, \mathsf{snd}(\langle x, y\rangle) = y, \mathsf{sdec}(k, \mathsf{senc}(k, m)) = m\}$$

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Let $\boldsymbol{\Sigma}$ be a order-sorted signature. A pair $\{s, t\}$ of terms $s, t \in \mathcal{T}_{\Sigma}(\mathcal{V})$ is called an equation, we write $s = t$.
The equational theory defined by E is the smallest congruence relation $=_E$ containing all instances of equations in E.

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Given a order-sorted term algebra $\mathcal{T}$, we define the set of all facts by

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A (labeled) multiset rewriting rule is a triple (p, a, c) of finite sequences $p, a, c \in \mathcal{F}^*$, written $p \,{-}[\, a \,]{\rightarrowtail}\, c$.

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Let Σ_{Fact} be an unsorted signature partitioned into linear and persistent fact symbols. Furthermore, assume there is a designated fact symbol $\mathsf{Fr} \in \Sigma_{\mathsf{Fact}}$ modelling freshness.

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$$\cup \, \{\mathsf{K}(x_1), \ldots, \mathsf{K}(x_k) \longrightarrow \mathsf{K}(f(x_1, \ldots, x_k)) \mid f \in \Sigma, \mathsf{arity}(f) = k\}$$

Traces and Security Properties

Given a multiset rewriting system R and a equational theory by E. This yields a transition relation $\Longrightarrow_{R,E}$ modelling the application of rewriting rules to multisets of facts.

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$$\mathrm{traces}_E(R) = \{[A_1, A_2, \ldots, A_n] \mid \exists S_1, \ldots, S_n : \emptyset \overset{A_1}{\Longrightarrow}_{R,E} S_1 \overset{A_2}{\Longrightarrow}_{R,E} \ldots \overset{A_n}{\Longrightarrow}_{R,E} S_n\}$$

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Security properties can then be formulated as first-order formulas on traces, e.g. secrecy properties:

$$\forall I, x : \mathsf{K}(x) \wedge \mathsf{Id}(I, x) \Rightarrow \mathsf{Corrupt}(I, x)$$

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Trace Formulas	Security properties (e.g. executability, secrecy, authenticity)

Tamarin-Prover in Practice

(Short) Demo ☺

Reference Implementation of IPSec

Building Blocks for IPSec

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rule gen_nonce:
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rule dh_calc:
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```
functions: prf/1
rule use_prf:
    let SKEYSEED = prf(<Ni, Nr, DH>)
    in
    [ State(Ni, Nr, DH) ] --> [ State(Ni, Nr, DH, SKEYSEED) ]
```

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```
rule use_aeenc:
let ct = senc(~secret, key_e)
    tag = mac(ct, key_a)
    hdr = < '120', ... >
in [ Fr(~secret), State(key_e, key_a) ] --> [ Out(<hdr, ct, tag>)]
```

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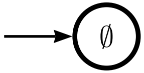
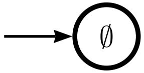
  - Signature schemes (cf. demo)
  - Authenticated encryption schemes (use EtA for now)
- Certificates (use identifier and signature(s) for now ✓)

Finite State Machine



Finite State Machine

Init Phase



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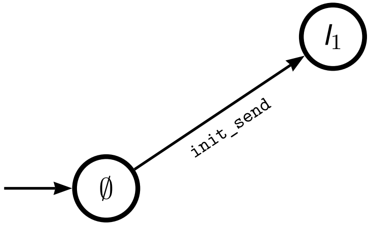
$$\begin{tikzcd} & I_1 \\ \arrow[r] & \emptyset \arrow[u, "\texttt{init\_send}"] \end{tikzcd}$$

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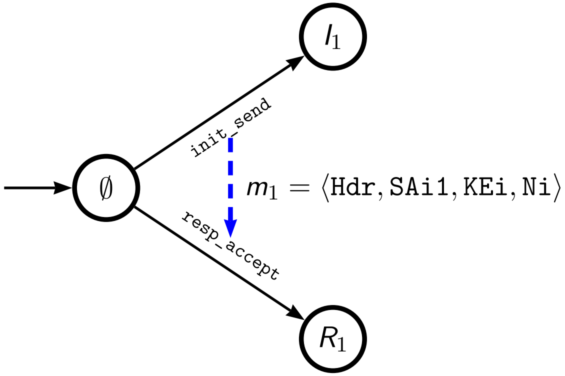

$\emptyset$
I_1
R_1
init_send
resp_accept
$m_1 = \langle \texttt{Hdr}, \texttt{SAi1}, \texttt{KEi}, \texttt{Ni} \rangle$

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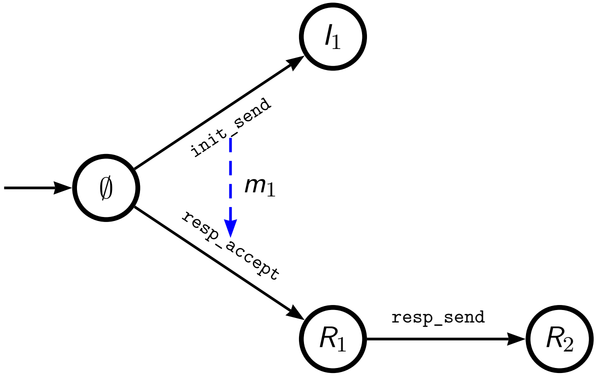

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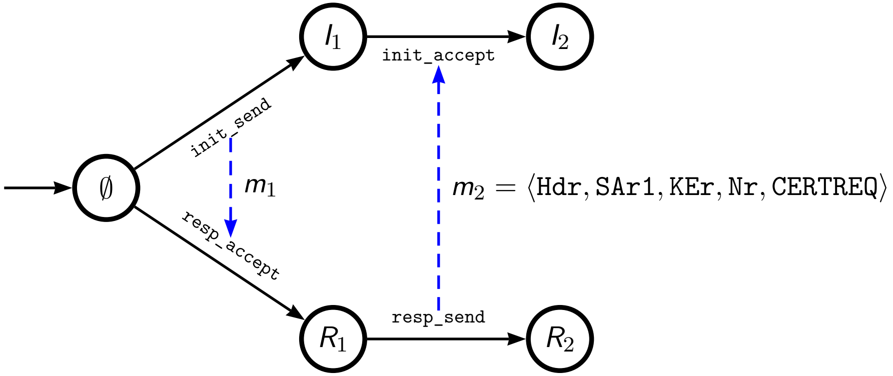
$\emptyset$
I_1
I_2
R_1
R_2
init_send
resp_accept
init_accept
resp_send
m_1
$m_2 = \langle \mathtt{Hdr}, \mathtt{SAr1}, \mathtt{KEr}, \mathtt{Nr}, \mathtt{CERTREQ} \rangle$

Finite State Machine

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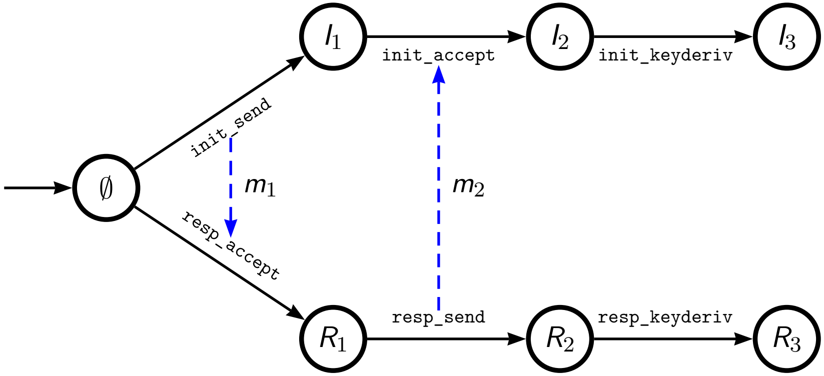

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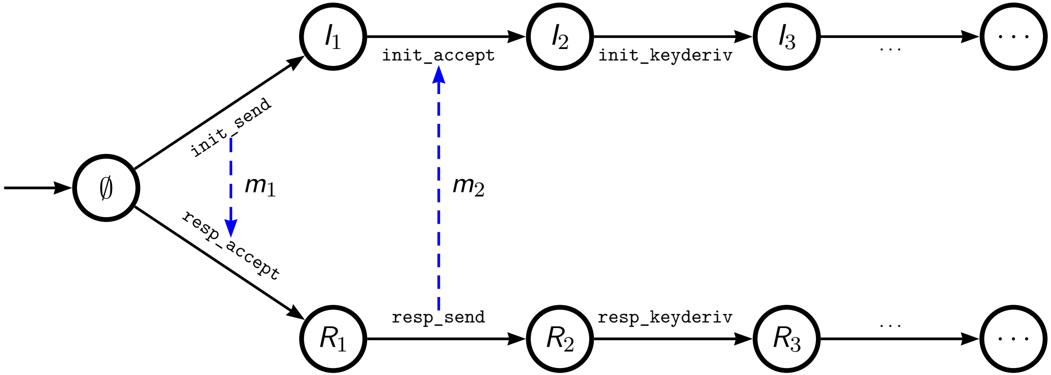

Code Walkthrough ☺

Lab-Goals Reflection

Goals for the Lab - Revisited

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- Theory of Tamarin-Prover

- Practical Application

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- Theory of Tamarin-Prover
  - mathematical foundation, in particular
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    - equational theories
    - operations: substitution, replacements, unification, matching, rewriting modulo equational theories
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References

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Thank you for your attention!